Implementation of a Feasible Control Design Process Incorporating Robustness Criteria for Wind-Excited High-Rise Buildings

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Abstract: In this paper, a control design process incorporating robustness criteria is developed specifically for wind-excited high-rise buildings and its feasibility is experimentally verified by implementing on a four degree-of-freedom scaled (1:300) model of a high-rise building through wind tunnel tests in along-wind and across-wind directions. For feasibility toward mature application, considerations in the control design process are made as practical and thorough as possible. The main features of the proposed design process include a suitable identification scheme that can realistically model wind loads and the possible interaction between control devices and structure, as well as a systematic way of incorporating robustness criteria in the controller for reducing tracking error, rejecting noise, attenuating disturbance, and maintaining stability in the presence of system uncertainty. To account for the complexity of wind load in system identification, wind spectrum is obtained through the high-frequency force-balance tests, and the corresponding state space wind model is constructed. The nominal system with the robustness criteria employed is transformed into a generalized $H_{\infty}$ control problem that can be further converted into a set of linear matrix inequalities (LMIs). Consequently, strictly proper output feedback $H_{\infty}$ controllers are thus determined by a simpler solution procedure in searching the solution of LMIs. As observed from experimental results, the performances of proposed $H_{\infty}$ controllers are quite remarkable and comparable to those of classical linear quadratic Gaussian (LQG) controllers. Additionally, the proposed controllers are numerically demonstrated to be more robust than classical LQG controllers under the existence of system uncertainty. This successful implementation shows that the design process and robust controllers proposed are feasible for wind-excited high-rise buildings and the resulting control performance is promising.

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Introduction

In the past decades, the new technique developed in civil construction has begot many more high-rise buildings in urban areas all over the world. The newly constructed Taipei 101 building in Taiwan with the total height of 509 m is one of the examples. Under wind excitation, the flexibility induced from their slender-ness might cause excessive displacements and accelerations on which building serviceability and human comfort depend. To effectively reduce such excessive responses, it has been well recognized from much literature that the use of structural active control serves as a good alternative.

As the progress of employing active control technique has largely come to the stage of actual implementation other than just numerical investigation in the interest of research, two typical issues concerning the control design process toward mature application shall be addressed. The first issue is regarding the accuracy of system prediction. It is well understood that system prediction plays a key role for ensuring good performance of the controllers. The system uncertainty resulted from prediction always exists but should be minimized as much as possible. In actual implementation of active control, one of the critical factors in system prediction, is mainly due to the integrated effect of active device (actuator) with structure, the so-called control-structure interaction (CSI), which was first systematically investigated by Dyke et al. (1995, 1996) on a scaled seismic-excited model and later confirmed by Wu (2000) using a full-scale seismic-excited building model with a total weight of 30 t. It was found in these studies that, due to such a CSI effect, the transfer function of the actuator is no longer characterized by a simple time delay but more complex dynamics, which involves system “zeros” that essentially comes from structural “poles.” That is, the actuators appear to have difficulty generating the active force at the frequencies of structural modes, and this difficulty grows as the scale of structure increases. Based on the previous experimental experiences of the writer’s (Wu 2000), not accounting for such an effect in the system identification degrades the performance and sometimes even causes the system to become unstable during control. Hence, a proper inclusion of CSI effect in the control design process during the implementation stage should be considerably inevitable. In the aspect of wind engineering application,
regardless of the difference of excitation sources, the same conclusion in this regard shall apply. In addition to the CSI effect, for wind-excited structures, system uncertainty can also come from the complexity of wind characteristics on structures, which can be best exemplified by the vortex shedding effect in the across-wind motion (Simiu and Scanlan 1996).

The second issue to be addressed is the profound consideration of robustness for controllers. The robustness criteria for controllers, according to modern control theories, involve the capability of maintaining performance robustness for reducing tracking error, rejecting noise, and attenuating disturbance, and maintaining stability robustness in the existence of system uncertainty (Zhou and Doyle 1998). Maintaining system stability in resistance to system uncertainty for controllers is relatively important because it directly links to structural safety, and its importance has been well recognized in the structural control society during the period that establishment of benchmark studies was in preparation (Balas 1998; Spencer et al. 1998; Young and Bienkiewicz 1998). In the newly announced wind-excited benchmark problem by Yang et al. (2004), an estimation error assumed in structural stiffness as the system uncertainty is compulsorily imposed on each controller to test its robustness. Therefore, accounting for all the robustness criteria in the controller design during implementation is important. Furthermore, among the advanced control strategies proposed in much of the literature (e.g., Kobori et al. 1998; Casciati 2002), $H_{\infty}$ control strategy is particularly useful because the robustness criteria can be mathematically interpreted as the $H_{\infty}$ norm of a transfer function smaller than an attenuation value (Glover et al. 1988; Doyle et al. 1989; Zhou and Doyle 1998). Hence, using the idea of $H_{\infty}$ control to account for robustness should be a proper choice.

To accommodate these two issues in implementation, in this paper, a control design process that contains a suitable identification scheme and a systematic procedure of incorporating all robustness criteria including the performance robustness and stability robustness using the theory of $H_{\infty}$ control strategy was developed specifically for control of wind-excited high-rise buildings. The proposed identification scheme involves identification for two subsystems, each is disturbed by its input source, i.e., the active force or wind load. For identifying Subsystem 1, the idea that was originally suggested by Dyke et al. (1995, 1996) for a scaled seismic-excited building model and later confirmed by Wu (2000) for a full-scale building model, was used. That is, a white noise command as an input to the actuator was used to excite the structure. As such, the control device and the structure were considered as a whole, and therefore the proper inclusion of the CSI effect to reduce the system uncertainty can be achieved. The identification of Subsystem 2 involves the state space modeling of the building and wind load that the former was constructed by extracting modal information from the identification results in Subsystem 1, while the latter used the results from a high-frequency force-balance test in a wind tunnel to better realize wind load complexity. The robustness criteria were incorporated with the nominal system and finally a generalized $H_{\infty}$ control problem was formulated. The solution method of the $H_{\infty}$ controller was obtained by suggesting the so-called linear matrix inequality (LMI)-based approach that not only makes the $H_{\infty}$ control theory more concise and straightforward but also facilitates the solution procedure (Gahinet 1992; Gahinet and Apkarian 1994; Chilali and Gahinet 1996; Scherer et al. 1997). This approach was developed by means of the recent progress in the numerical tool of LMI. Herein, to simplify the $H_{\infty}$ controller design, strictly proper controllers with dynamic output feedback are used and the solutions are computed by means of a simpler version of LMI-based solution procedure proposed in Gahinet (1992).

The proposed control design process and robust controllers thus designed for a four degree-of-freedom scaled (1:300) high-rise building model equipped with an active mass driver was implemented through wind tunnel tests to verify its feasibility and applicability to wind-excited high-rise buildings. Since wind tunnel tests on an elastic building model can best represent actual situations that uncertainties exist, the performance in the experimental results reflects the performance robustness of the controller used in the presence of uncertainties. The performances of proposed robust $H_{\infty}$ controllers in the wind-induced along-wind and across-wind motions were presented, and comparisons were made with those from classical linear quadratic Gaussian (LQG) controllers. Additionally, uncertainty tests for stability using numerical simulation for both controllers were also conducted and thus their stability robustness in resistance to uncertainty is demonstrated.

**Structural Model and Experimental Setup**

To realistically represent high-rise buildings, a scaled building model based on the 76-story wind-excited benchmark building proposed by the ASCE structural control committee (Yang et al. 2004) was constructed for the experiments. To account for the possible multiple mode responses, the 76-story prototype is condensed to a four degree-of-freedom (DOF) shear type model, and then scaled down according to the scaling factors of 1:300, 1:6, and 1:1 in length, velocity, and density, respectively (accordingly, 1:50 in time). Fig. 1(a) shows the frame skeleton of the scaled model. The completed model with the exterior walls at the atmospheric boundary layer wind tunnel at Tamkang Univ., Taiwan is shown in Fig. 1(b). It has a height/width ratio of 6.4, which is susceptible to wind disturbance.

The active control device installed on the top floor of the building model is an active mass driver (AMD) system, which is composed of a linear servo-motor and a moving mass, as shown in Fig. 2. The maximum displacement (stroke) allowed in the motor is limited to 1 cm. During experiments, the building responses, i.e., the absolute displacements and absolute accelerations of all DOFs, the absolute acceleration of the moving mass of the AMD system and the control command were recorded. The notation for the DOFs was numbered from the bottom to the top. Among them, the displacements and accelerations of the first, second, and fourth DOFs were denoted as the controlled output vector $z=[x_1, x_2, x_4, \dot{x}_1, \dot{x}_2, \dot{x}_4]^T$ to be used in the identification later. The feedback quantities (e.g., absolute acceleration herein)
during active control can be extracted from the controlled output \( z \) and denoted as the measured output \( y \).

In the wind tunnel tests, the atmospheric boundary layer for suburban area according to ANSI A58.1 (Terrain Classification B) was simulated, resulting in a gradient height of 400 m and power law exponent of the mean wind velocity equal to 0.23. Some other detailed description of this scaled building model with the AMD system can be found in Wu and Pan (2002).

Formulation of Identification Scheme

The control-integrated wind-excited system was considered as a linear system that can be decomposed into two subsystems as shown in Fig. 3(a): one is due to the excitation of active force exerted from the active mass driver, denoted as Subsystem 1, while the other is due to wind force disturbance, denoted as Subsystem 2. In each subsystem, the identification can be performed independently by using the output/input relation. The Laplace domain realization of the control-integrated wind-excited system in terms of transfer functions is shown in Fig. 3(b). The identification process of each subsystem is described in detail as follows.

Identification of Subsystem 1

For identifying Subsystem 1, the idea originally suggested by Dyke et al. (1995, 1996) was adopted, i.e., a scalar command \( U \) was used as the input source to be fed into the AMD system which the active force is exerted from. In the experiments, a set of band-limited white noise command \( U \) (in volts) was used and the frequency response functions (FRFs) of the quantities in \( z \) versus \( U \), i.e., \( H_{11}(j\omega) \), were measured and computed. The dotted curves shown in Fig. 4 are the FRFs of the displacements and accelerations of the first and fourth DOFs. The mathematical form of the components of \( H_{11}(j\omega) \) can be obtained by curve fitting each experimental data to a ratio of two polynomials in \( j\omega \) expressed as

\[
\frac{[b_{n} (j\omega)^{n} + b_{n-1} (j\omega)^{n-1} + \ldots + b_{1} (j\omega) + b_{0}]}{[(j\omega)^{m} + a_{m-1} (j\omega)^{m-1} + \ldots + a_{1} (j\omega) + a_{0}]} , n \leq m,
\]

in which the coefficients \( b_{n}, b_{n-1}, \ldots, b_{1}, a_{m-1}, \ldots, a_{1}, a_{0} \) are determined by the weighted least-square-error method (e.g., Wu 2000; Wu and Pan 2002). By keeping the denominators the same, the fitted results \( n=10, m=14 \) for displacements and \( n=12, m=14 \) for accelerations) are shown by the solid curves in Fig. 4. Consequently, these fitted FRFs can be further converted into a state space equation in time domain expressed by
\[ \begin{align*}
Z_t &= A_t Z_t + B_t U \\
\dot{z}_t &= C_t Z_t + D_t U
\end{align*} \]

in which \( Z_t \) = state vector; \( z_t \) = part of \( z \) contributed from \( U \); and \( A_t, B_t, C_t, D_t \) = matrices with appropriate dimensions. Since Eq. (1) represents the dynamics from the AMD command to building responses, it may have contained the possible interaction between the active control device and the structure.

**Identification of Subsystem 2**

As shown in Fig. 3(a), the block of Subsystem 2 contains two models: the building model and wind load model which are identified separately as follows.

**Building Model**

Since the measurements of responses due to AMD action are handy from the identification results in Subsystem 1, the modal properties can be extracted from them for constructing the building model. From structural dynamics, the dynamics of a building model can be described by the equation of motion expressed by

\[ M_S \ddot{X}_S(t) + C_S \dot{X}_S(t) + K_S X_S(t) = F(t) \]

where \( M_S, C_S, K_S \) are mass, damping, and stiffness matrices of the building model and \( F(t) \) = wind force vector. While the mass of each DOF in the test was directly measured, in reality the mass can be best estimated using the finite element approach. From the measured data in subsystem 1, the FRFs of the accelerations of all DOFs versus the inertial force of the moving mass can be computed. Since the active force exerted from the AMD is the inertial force of the moving mass, these FRFs thus obtained represent the response outputs from the input force acting on the top DOF. The curve shapes of these FRFs are similar to those shown in Fig. 4 because the command \( U \) is set to track the acceleration of the building model.

In the same manner, the state space equation can be formed after curve fitting these FRFs, i.e.,

\[ \begin{align*}
\dot{Z}_4(t) &= A_4 Z_4(t) + B_4 F_i(t) \\
\dot{z}_4(t) &= C_4 Z_4(t) + D_4 F_i(t)
\end{align*} \]

in which \( F_i(t) \) = active force acting on the fourth DOF. Note that now the \( z_4 \) in Eq. (3) contains the accelerations of all four DOFs, i.e., \( z_4 = [\dot{x}_1, \dot{x}_2, \dot{x}_3, \dot{x}_4]^T \). Thus, the modal decomposition of this state Eq. (3) can be performed to provide information of modal properties. First, by assuming small (or proportional) damping in the building, the \( j \)th pair eigenvalue of the system matrix \( A_4 \) shall have the form of \(-\xi_j \omega_j \pm i \sqrt{1-\xi_j^2} \omega_j \), in which \( \xi_j \) and \( \omega_j \) are the \( j \)th mode damping ratio and natural frequency. The resulting natural frequencies and damping ratios are 6.32, 31.57, 62.97, and 131.23 Hz, and 2.7, 2.41, 3.53, and 2.22%, respectively. Second, by substituting the \( j \)th pair eigenvector of \( A_4 \) into the state vector \( Z_4 \) and letting \( F_i \) be equal to zero (i.e., free vibration) in Eq. (3), the complex-valued controlled output vector \( \dot{z}_4 \) shall represent the \( j \)th mode shape of the building. Due to small (or proportional) damping, the complex mode shape can be converted to a real one because the components in \( \dot{z}_4 \) are either in phase or out of phase. This is based on the fact that, for free vibration with zero initial conditions, the contribution of the \( j \)th pair of modes consists of the vibrational mode shape multiplied by a time-varying function \( \exp(-\xi_j \omega_j t) \sin(\sqrt{1-\xi_j^2} \omega_j t) \). Finally with the damping ratios, natural frequencies, mode shapes, and mass matrix \( M_S \) given, the damping and stiffness matrices \( C_S \) and \( K_S \) in Eq. (2) can be obtained by following the technique from modal analysis. Consequently, the state space realization in this case can be converted from Eq. (2) and expressed as

\[ \begin{align*}
\dot{Z}_b &= A_b Z_b + B_b F \\
\dot{z}_b &= C_b Z_b + D_b F \\
Z_b &= \begin{bmatrix} X_S \\ \dot{X}_S \end{bmatrix} \\
A_b &= \begin{bmatrix} I & 0 \\ -M_S^{-1} K_S & -M_S^{-1} C_S \end{bmatrix} \\
B_b &= \begin{bmatrix} 0 \\ M_S^{-1} \end{bmatrix} \\
C_b &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
D_b &= \begin{bmatrix} 0 \\ B_b(5,6,8) \end{bmatrix}
\end{align*} \]

In Eq. (4), \( z_b \) represents the part of \( z \) contributed from the wind force \( F \); the notation \( A_b(5,6,8) \) and \( B_b(5,6,8) \) in Eq. (5) means the matrices consisting of the fifth, sixth, and eighth rows of \( A_b \) and \( B_b \), respectively.

**Wind Load Model**

In reality, the wind forces are usually not accessible in field. Thus, the estimation through wind tunnel tests becomes a possible option from the engineering point of view. In wind engineering, the so-called high-frequency force-balance test is the most practical method to acquire wind load data so far, and it has been widely used as a standard test to find the design wind load for a building at the design stage. This test is designed to be conducted on a rigid building model that has the same geometry shape as the elastic model. It is based on the fact that the base shear force measured by the force-balance transducer is in equilibrium with the total horizontal wind force on the building due to the high rigidity of the test model. It is noted that the wind load thus measured does not contain the part interacted with the deformed structure due to the high rigidity of the test model. Hence, it may not be very precise for situations with large deformation, such as the across-wind load. However, from the control perspective, a possible way to overcome this is to treat it as a system uncertainty in the robustness consideration for the controller design.

On the conservative side, it is conceivable to estimate the autopower spectrum of each wind force, \( S_{F_i} \), by plorating out the power spectrum of the base shear force following the profile of mean wind velocity square multiplied by the tributary area. To further facilitate the controller design, it is necessary to construct a state space wind load model to simulate the wind forces induced by the excitation of a fictitious white noise \( W \) as shown in Fig. 3(a). For simplicity, spatially complete corrections among the wind forces were assumed herein, although spatially partial correlations are more realistic. Thus, the auto-power spectrum of
each wind force, $S_{F_{i}F_{j}}$ can be decomposed into the frequency response function from $W$ to $F_{i}$, i.e., $H_{F_{i}W}(\omega)$, from the relation

$$|H_{F_{i}W}(\omega)| = \sqrt{S_{F_{i}F_{j}}(\omega)}$$

in which a fictitious white noise $W$ with an unitary spectrum intensity is taken without loss of generality. The dotted curves shown in Fig. 5 are the plots of $|H_{F_{i}W}(\omega)|$ and $|H_{F_{j}W}(\omega)|$ in the along-wind and across-wind directions when the mean wind velocity is equal to 8 m/s at the building height. Second, by the curve-fitting technique that keeps the denominators of $H_{F_{i}W}(\omega)$ the same, the mathematical form of the frequency response functions can be obtained using the orders of $n=2$ and $m=3$. The fitted curves of $H_{F_{i}W}(\omega)$ and $H_{F_{j}W}(\omega)$ for along-wind and across-wind directions are shown in Fig. 5 as denoted by the solid curves. Note that the frequency at the peaks shown in Figs. 5(c) and 5(d) represents the vortex-shedding frequency. Thus, the resulting state space equation of the wind load model can be obtained and expressed as

$$\dot{\mathbf{f}} = \mathbf{A}_f \mathbf{f} + \mathbf{B}_f W$$

$$\mathbf{F} = \mathbf{C}_f \mathbf{f} + \mathbf{D}_f W$$

in which $\mathbf{f}$=three-dimensional state vector of the wind load model; $W$=scalar fictitious white noise with an unitary spectrum; and the dimensions of $\mathbf{A}_f$, $\mathbf{B}_f$, $\mathbf{C}_f$, and $\mathbf{D}_f$=$(3 \times 3)$, $(3 \times 1)$, $(4 \times 3)$, and $(4 \times 1)$, respectively.

Consequently, the building Eq. (4), and the wind load model Eq. (7) were cast into a state equation for Subsystem 2 expressed by

$$\dot{\mathbf{Z}}_W = \mathbf{A}_W \mathbf{Z}_W + \mathbf{B}_W W$$

$$\mathbf{Z}_W = \mathbf{C}_W \mathbf{Z}_W + \mathbf{D}_W W$$

$$\mathbf{Z}_W = \begin{bmatrix} \mathbf{Z}_b \\ \mathbf{f} \end{bmatrix}$$

in which $\mathbf{Z}_W$ is the part of $\mathbf{z}$ contributed from $W$.

**Control-Integrated Wind-Excited System**

With the Subsystems 1 and 2 identified, the control-integrated wind-excited system was finally determined by superposing Eqs. (1) and (8), i.e.

$$\dot{\mathbf{z}}_a = \mathbf{A}_a \mathbf{z}_a + \mathbf{B}_a U + \mathbf{E}_a W$$

$$\mathbf{z} = \mathbf{z}_U + \mathbf{z}_W = \mathbf{C}_a \mathbf{z} + \mathbf{D}_a U + \mathbf{F}_a W$$

$$\mathbf{D}_a = \mathbf{D}_W$$

$$\mathbf{F}_a = \mathbf{D}_W$$

$$\mathbf{Z}_a = \begin{bmatrix} \mathbf{Z}_U \\ \mathbf{Z}_W \end{bmatrix}$$

$$\mathbf{A}_a = \begin{bmatrix} \mathbf{A}_f & 0 \\ 0 & \mathbf{A}_W \end{bmatrix}$$

$$\mathbf{B}_a = \begin{bmatrix} \mathbf{B}_U \\ 0 \end{bmatrix}$$

$$\mathbf{E}_a = \begin{bmatrix} 0 \\ \mathbf{B}_W \end{bmatrix}$$

$$\mathbf{C}_a = \begin{bmatrix} \mathbf{C}_U & \mathbf{C}_W \end{bmatrix}$$

**Model Reduction and Nominal System**

The control-integrated wind-excited system in Eq. (9), referred to as the full-order system, can be further reduced to a minimal system using the balanced state reduction method presented in Moore (1981). By this method, Eq. (9) is first transformed into a balanced-state system in which the controllability and observability grammians are equal and diagonalized. The value of the diagonal element is called the Hankel singular value which represents the relative controllability (and observability) of the corresponding state. After truncating the states with relatively small Hankel singular values, an eight-state reduced-order system represented by the matrices $(\mathbf{A}_p, \mathbf{B}_p, \mathbf{C}_p, \mathbf{D}_p, \mathbf{E}_p, \mathbf{F}_p)$ was constructed. The typical frequency response functions of $x_4$ and $\dot{x}_4$ in the reduced-order and full-order systems, and the corresponding reduction errors are shown in Figs. 6 and 7 for along-wind and across-wind motions, respectively. As shown in Figs. 6 and 7, the first three structural modes are reasonably preserved. This reduced-order system was finally used as the nominal system for the design of the following robust $H_\infty$ controller.
Formulation of Generalized $H_\infty$ Control Problem

A block diagram of a physical system with active control is illustrated by the blocks in solid line portions shown in Fig. 8, in which $P$=nominal system; $K$=controller system; $r$=reference signal; $U$=control command generated from the controller; $W$=excitation; $n$=measurement noise; $y$=measured response; and $e$=error signal between the measured response and the reference. Herein the reference signal is taken as $r=0$ because the control objective is to suppress vibration. Meanwhile, the measured output is taken as $y=[x_1 \ x_2 \ x_3]^T$, which can be extracted from the controlled output $z$. As learned from the modern control theory, the robustness criteria, namely, the performance robustness of reducing tracking error, rejecting noise and attenuating disturbance, and stability robustness with respect to system uncertainty (Zhou and Doyle 1998) can be considered equivalent to finding a controller $K(s)$ for a generalized plant $G(s)$, under the proper choice of weighting matrices $W_x$, $W_U$, and $W_y$, such that $\|H_{e,d}\|_\infty < \gamma$, in which $H_{e,d}$=transfer function from $d$ to $z_e$, and $\gamma$=so-called attenuation value. The generalized plant $G(s)$ is converted from the physical system diagram in Fig. 8 to form a generalized controlled output $z_c=[x_c^T \ z_{WU}^T z_y^T]^T$, a generalized measured output $e$, and a generalized exogenous input $d=[n^T \ W^T]^T$. This is called the generalized $H_\infty$ control problem as shown in Fig. 9. It should be noted that: (1) the components $x_c$, $z_{WU}$, and $z_y$ in $z_c$ can be considered as the weighted $e$, $U$, and $y$, respectively, as shown by the dotted line portion in Fig. 8; and (2) a smaller value of $\gamma$ can guarantee stronger robustness. In the following, the systematic way of constructing the state equation for the generalized plant $G(s)$ is described.
Construction of Generalized Plant

Following the blocks in Fig. 8, the state equations of the nominal system \( P(s) \), weightings \( W_e(s) \), \( W_u(s) \), and \( W_y(s) \) can be expressed as

\[
\dot{z}_p = A_p z_p + B_p u + E_p w \\
y = C_p z_p + D_p u + F_p w \\
\dot{z}_e = A_e z_e + B_e e \\
z_e = C_e z_e + D_e e \\
\dot{z}_{wu} = A_{wu} z_{wu} + B_{wu} u \\
z_{wu} = C_{wu} z_{wu} + D_{wu} u \\
\dot{z}_y = A_y z_y + B_y y \\
z_y = C_y z_y + D_y y
\]

respectively, in which \( z_p, z_e, z_{wu}, z_y \) = state vectors; and \( A_p, B_p, C_p, D_p, E_p, F_p, A_e, B_e, C_e, D_e, A_{wu}, B_{wu}, C_{wu}, D_{wu}, A_y, B_y, C_y, \) and \( D_y \) = constant matrices with appropriate dimensions. By considering \( n \) and \( W \) as the exogenous input \( d \); \( z_e, z_{wu}, \) and \( z_y \) as the generalized controlled output \( z_e \); and \( e \) as the generalized measured output, the generalized plant \( G(s) \) to be controlled can be written as the state equations expressed by

\[
\dot{z} = Az + B_1 d + B_2 u \\
z_e = C_1 z + D_{11} d + D_{12} u \\
e = C_2 z + D_{21} d + D_{22} u
\]

in which

\[
Z = \begin{bmatrix} z_p \\ z_e \\ z_{wu} \\ z_y \end{bmatrix} \\
d = \begin{bmatrix} n \\ W \end{bmatrix} \\
z_e = \begin{bmatrix} z_e \\ z_{wu} \\ z_y \end{bmatrix}
\]

\[
A = \begin{bmatrix} A_p & 0 & 0 & 0 \\ -B_p C_p & A_e & 0 & 0 \\ 0 & 0 & A_{wu} & 0 \\ B_p C_p & 0 & 0 & A_y \end{bmatrix} \\
B_1 = \begin{bmatrix} 0 & E_p \\ -B_e & -B_e F_p \\ 0 & 0 \\ 0 & B_e F_p \end{bmatrix} \\
B_2 = \begin{bmatrix} B_p \\ -B_p D_p \\ B_{wu} \\ B_p D_p \end{bmatrix} \\
C_1 = \begin{bmatrix} -D_e C_p & C_e & 0 & 0 \\ 0 & 0 & C_{wu} & 0 \\ D_y C_p & 0 & 0 & C_y \end{bmatrix} \\
D_{11} = \begin{bmatrix} -D_e & -D_e F_p \\ 0 & 0 \\ 0 & D_e F_p \end{bmatrix} \\
D_{12} = \begin{bmatrix} -D_p D_p \\ -D_{wu} \\ D_p D_p \end{bmatrix} \\
C_2 = \begin{bmatrix} -C_p & 0 & 0 & 0 \end{bmatrix} \\
D_{21} = \begin{bmatrix} -I & -F_p \end{bmatrix} \\
D_{22} = -D_p
\]

in which \( e = -y - n \) = generalized measured output used as the feedback quantities for control.

Linear Matrix Inequality-Based Solution Procedure

The recent development on LMIs has facilitated a more concise and straightforward formulation for the generalized \( H_\infty \) control problem shown in Fig. 9 (e.g., Gahinet 1992; Gahinet and Apkarian 1994; Chilali and Gahinet, 1996; Scherer et al. 1997). In a case where strictly proper controllers are to be designed, a simpler solution procedure presented in Gahinet (1992) is suggested herein to demonstrate the convenience of such a LMI-based approach. Since the derivation involves tedious proofs that are beyond the scope of this research, only the rationale and concluded solution procedures will be described in the following, while the formulation is referred to in the Appendix.

Let the dimension of \( A \) in the generalized plant system be \( N \times N \) and a full-order strictly proper controller \( K(s) \) be expressed in the state space as

\[
\dot{z}_k = A_k z_k + B_k e \\
U = C_k z_k
\]

in which \( z_k \) = state vector of the controller; \( A_k, B_k, C_k \) = matrices to be determined; and \( A_k \) has the same di-
Compute the full-rank matrices $M$, $D$, and $C$ for a simpler LMI-based approach are specifically summarized as follows:

1. Use Matlab LMI Toolbox to find solutions for the matrices $R$, $S$, $B$, and $C$ by constructing a minimization problem with an objective function expressed by

$$ J = \gamma + \alpha \cdot \text{Tr}(R) + \beta \cdot \text{Tr}(S) $$

subject to three LMI constraints described in Inequalities (24), (30), and (31). Using the objective function of Eq. (17) is for the sake of finding a minimum attenuation value $\gamma$ because a smaller $\gamma$ can guarantee stronger robustness. The modulations of the trace of $R$ and $S$ are also included in Eq. (17) because smaller traces of $R$ and $S$ are helpful in slowing down the frequency response of the controller.


3. Solve $A$ from $\Phi = 0$ [see Eq. (28)].

4. Solve $B$, $C$, and $A$ (in this order) from Eq. (29).

5. Convert the controller equation in terms of $A$, $B$, and $C$ with $e$ as the measured output into that in terms of $A$, $B$, and $C$ with $e$ as the measured output by using the loop shifting technique, i.e.

$$ A = \tilde{A} - BD_2C $$

$$ B = \tilde{B} $$

$$ C = \tilde{C} $$

Discretization of Controller

Once the controller expressed by Eq. (16) is determined, it should be discretized before implementation. The sampling time used for the controller in the tests is 0.001 s.

Results of Wind Tunnel Tests

Wind tunnel tests of the high-rise building model are conducted using the robust $H_\infty$ controllers designed by the LMI-based approach under two different mean wind velocities of 8.0 and 8.8 m/s at the building height. These velocities are equivalent to the reference wind velocities (10 m height) of 21.8 and 24 m/s for the prototype building in the suburban area with a gradient height of 400 m and power law exponent of 0.23. Both the along-wind and across-wind loadings are tested on the building to investigate the performance of active control in each direction. Following the identification scheme proposed, the controller-integrated wind-excited system was constructed and thus the nominal system was obtained. For identifying Subsystem 2, the spatially completely correlated wind model [Eq. (7)] based on the force-balance test result under a mean wind speed of 8 m/s was used. The robustness criteria were then incorporated with the nominal system following the procedure presented to form a generalized $H_\infty$ problem, and a LMI-based solution procedure was performed to find the $H_\infty$ controllers. The design parameters for the robust $H_\infty$ controller are as follows: (1) for the along-wind direction, the weightings are $\alpha = \beta = 10^{-8}$, $W_2(s) = (7.21s + 179.89)/(s^2 + 0.8s + 1562.3)$, $I_{3 \times 3}$, and $W_3(s) = (2.25s + 0.8)/(s^2 + 0.8s + 1562.3)$; (2) for the across-wind direction, $\alpha = \beta = 10^{-8}$, $W_2(s) = (0.02s + 10)/(2.5s + 500)$, and $W_3(s) = (25.2s + 980)/(s + 4000)$.

Table 1. Experimental Results of Response Quantities without Control

<table>
<thead>
<tr>
<th>Degree of freedom (DOF)</th>
<th>Along wind</th>
<th>Across wind</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Peak</td>
<td>RMS</td>
</tr>
<tr>
<td><strong>Displacement (mm)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.3620</td>
<td>0.0907</td>
</tr>
<tr>
<td>2</td>
<td>0.2904</td>
<td>0.0799</td>
</tr>
<tr>
<td>4</td>
<td>0.2624</td>
<td>0.0709</td>
</tr>
<tr>
<td><strong>Acceleration (g)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.0593</td>
<td>0.0155</td>
</tr>
<tr>
<td>2</td>
<td>0.0632</td>
<td>0.0166</td>
</tr>
<tr>
<td>4</td>
<td>0.0646</td>
<td>0.0169</td>
</tr>
<tr>
<td>Maximum($\bar{u}_{mdl}$)</td>
<td>0.0669g</td>
<td></td>
</tr>
<tr>
<td><strong>Displacement (mm)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.4609</td>
<td>0.1207</td>
</tr>
<tr>
<td>2</td>
<td>0.4024</td>
<td>0.1106</td>
</tr>
<tr>
<td>4</td>
<td>0.3730</td>
<td>0.0978</td>
</tr>
<tr>
<td><strong>Acceleration (g)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.0712</td>
<td>0.0206</td>
</tr>
<tr>
<td>2</td>
<td>0.0787</td>
<td>0.0222</td>
</tr>
<tr>
<td>4</td>
<td>0.0748</td>
<td>0.0224</td>
</tr>
<tr>
<td>Maximum($\bar{u}_{mdl}$)</td>
<td>0.1827g</td>
<td></td>
</tr>
</tbody>
</table>
As a result, the minimized attenuation value $\gamma$ is $10.6672$ for the along-wind motion, and $1.1388$ for the across-wind motion. The peak and root-mean-square (RMS) values of building responses within 60 s without control under mean wind speed of 8 and 8.8 m/s are shown in all the columns of Table 1, respectively. In Table 1, $\ddot{x}_{nd}$ represents the absolute acceleration of the moving mass in the AMD system. The response quantities using the robust LMI-based $H_\infty$ controller were tabulated in Table 2; those in the first part are for the case under mean wind speed of 8.0 m/s while those in the second part are for that under mean wind speed of 8.8 m/s. In the tables throughout the paper, the values inside the parentheses are the percentages of reduction with respect to the responses without control. As demonstrated in Table 2, the performance of the LMI-based $H_\infty$ controller can achieve above 50% in reducing the peak and rms values of the displacement and accelerations in both the along-wind and across-wind directions even under different wind speeds of excitation.

The experiments using the classical LQG controllers were also conducted for comparison. These LQG controllers were designed based on the same nominal system $(A_P, B_P, C_P, D_P, E_P, F_P)$, and the detail formulation of LQG controllers is referred to the Appendix in Wu (2000). Following the notation in Wu (2000), the design parameters for the LQG controller are listed as follows: (1) for the along-wind motion: $Q = \text{diag}(1 \text{ mm}^{-1}, 1 \text{ mm}^{-1}, 1 \text{ mm}^{-1}, 10^4 \text{ g}^{-1}, 10^4 \text{ g}^{-1}, 10^4 \text{ g}^{-1})$, $R = 10^{-5} \text{ V}^{-1}$, $\bar{S}_{WW} = 10^{-17} \text{ N}^{-2}$, $\bar{S}_{\gamma\gamma} = 10^{-10} I_{3 \times 3} \text{ g}^{-1}$; (2) for the across-wind motion: $Q = \text{diag}(1 \text{ mm}^{-1}, 1 \text{ mm}^{-1}, 1 \text{ mm}^{-1}, 10^3 \text{ g}^{-1}, 10^3 \text{ g}^{-1}, 10^2 \text{ g}^{-1})$, $R = 30 \text{ V}^{-1}$, $\bar{S}_{WW} = 10^{-17} \text{ N}^{-2}$, $\bar{S}_{\gamma\gamma} = 10^{-10} I_{3 \times 3} \text{ g}^{-1}$. The building responses using the LQG controller in both along-wind and across-wind directions under two different wind speeds were listed in all the columns of Table 3, respectively. As shown in the parentheses in Table 3, the response reductions of the classical LQG controller can be determined from the feedback quantities is contained in the classical LQG controller that was designed without considering a filter. The same result was observed in Wu and Pan (2002). The performances of the controllers in the experimental results can

<table>
<thead>
<tr>
<th>Degree of freedom (DOF)</th>
<th>Along wind</th>
<th>Across wind</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Peak</td>
<td>RMS</td>
</tr>
<tr>
<td>Mean wind speed=8.0 m/s</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Displacement (mm)</td>
<td>1</td>
<td>0.1999 (44.77)</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.1346 (53.65)</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.1307 (50.19)</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0.0321 (45.80)</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.0357 (43.48)</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.0326 (49.55)</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0.2550 (44.67)</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.1963 (51.22)</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.1823 (51.13)</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0.0535 (24.86)</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.0420 (46.59)</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.0442 (40.90)</td>
</tr>
<tr>
<td>Acceleration (g)</td>
<td>1</td>
<td>Maximum($\ddot{x}_{nd}$)=0.3121g</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>Maximum($\ddot{x}_{nd}$)=0.1410 V</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>Maximum($\ddot{x}_{nd}$)=0.0928</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>Maximum($\ddot{x}_{nd}$)=0.0326</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>Maximum($\ddot{x}_{nd}$)=0.0357</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>Maximum($\ddot{x}_{nd}$)=0.0326</td>
</tr>
<tr>
<td>Acceleration (g)</td>
<td>1</td>
<td>Maximum($\ddot{x}_{nd}$)=0.2550</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>Maximum($\ddot{x}_{nd}$)=0.1963</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>Maximum($\ddot{x}_{nd}$)=0.1823</td>
</tr>
<tr>
<td>Mean wind speed=8.8 m/s</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
reflect the performance robustness in the presence of uncertainties because wind tunnel tests on the elastic building model can best represent actual situations that uncertainties exist. Therefore, as demonstrated by the results in Tables 2 and 3, the performance robustness of both \( H_\omega \) and classical LQG controllers are fairly equal in the presence of all possible uncertainties.

To make a further comparison in the aspect of stability robustness for the classical LQG controllers and the proposed \( H_\omega \) controllers, an uncertainty test for stability using numerical simulation was conducted. Conducting uncertainty tests for stability in experiments was not possible because the true system is hard to define, and therefore there was no comparison basis to define the “size” of system uncertainty. However, in these numerical tests, the identified control-integrated wind-excited system (under mean wind speed of 8 m/s) can be considered as the true system, and an additional system uncertainty expressed as a transfer function \( \delta(s+1)/(s^2+5s+1,400) \), in which \( \delta \) is a scalar, was superposed to the true system when numerical simulations using the \( H_\omega \) controllers and LQG controllers were conducted. Such a transfer function was used because it has a dynamics with a single mode at 6 Hz and damping ratio 6.7%, which can represent the system uncertainty that possibly occurs with the first structural mode emphasized. Note that the controller was designed based on the nominal system, therefore in the simulation the total uncertainty actually contains this additional uncertainty and reduction errors shown in Figs. 6 and 7 as well. In the along-wind and across-wind motions, as the scalar \( \delta \) is increased to 1.2 and 16.8, respectively, as shown in Figs. 10(a and b), the building responses using the \( H_\omega \) controller remain stable, however those from the LQG controller become unstable, as shown in Figs. 10(c–f). It is noted that the response in the along-wind motion [Fig. 10(d)] oscillates in a very slow frequency but grows gradually to infinity. From these tests, it was demonstrated that because the robustness criteria were considered in the design of the \( H_\omega \) controller, it is superior to the classical LQG controller in maintaining stability robustness.

### Conclusions

In this paper, a control design process incorporating robustness criteria was developed specifically for wind-excited high-rise buildings and its feasibility was experimentally verified by implementing on a four degree-of-freedom scaled building model.
through wind tunnel tests in along-wind and across-wind directions. To overcome the uncertainty effect, the control design process proposes a suitable identification scheme that can realistically model wind loads and the possible CSI effects, as well as a systematic procedure of incorporating robustness criteria in the controller design for maintaining performance robustness and stability robustness. Thus, the nominal system with the robustness criteria employed was converted into a generalized $H_\infty$ control problem, and the convenience of the LMI-based solution procedure was also verified from this implementation. The experimental results of using classical LQG controllers were also presented for comparison.

From the experimental results, the following conclusions were made:

1. The control design process and LMI-based $H_\infty$ controllers proposed were successfully implemented on a wind-excited high-rise building model through wind tunnel tests. The experimental results verified the feasibility and applicability of this approach to wind-excited buildings.

2. The performance of the LMI-based $H_\infty$ controller is comparable to that of the LQG controller that was designed based on the same nominal system. As demonstrated by the remarkable response reduction in both the along-wind and across-wind motions of the experimental results, both controllers have fairly equal performance robustness under uncertainties of wind speed variation and other possible prediction errors.

3. Based on the numerical uncertainty tests, the LMI-based $H_\infty$ controller that was designed by considering robustness criteria is more robust than the classical LQG controller for both along-wind and across-wind motions in maintaining structural stability in resistance to system uncertainty.

### Acknowledgment

The writers wish to express their gratitude to the National Science Council of Taiwan for financial support under Grant No. NSC 91-2211-E-032-017.

### Appendix. Derivation of Linear Matrix Inequality-Based $H_\infty$ Controller

For simplicity of derivation, another measured output $\bar{e}=e-D_2\bar{Z}+D_2d$ instead of $e$ in Eq. (14) is used first, while this intermediate step will be eliminated by loop shifting described in Eq. (18). Thus, the corresponding notations $A_K$, $B_K$, $C_K$, and $e$ in Eq. (16) are replaced by $\bar{A}_K$, $\bar{B}_K$, $\bar{C}_K$, and $\bar{e}$, respectively. The bounded real lemma (Zhou and Doyle 1998) states that having the control constraint $|H_{e,d}|<\gamma$ achieved is equivalent to satisfy the statement that there exists a $N\times N$ symmetric positive definite matrix $X$ (denoted as $X>0$) such that

$$
\begin{bmatrix}
A_d^T X + X A_d & X B_d & C_d^T \\
B_d^T X & -\gamma I & D_d^T \\
C_d & D_d & -\gamma I
\end{bmatrix} < 0
$$

or in an equivalent form [by using the Schur complement (Zhou and Doyle 1998)] expressed as

$$
A_d^T X + X A_d +
\begin{bmatrix}
B_d^T X & \gamma I & -D_d^T \\
C_d & -\gamma I & D_d \\
D_d^T & C_d & -\gamma I
\end{bmatrix} < 0
$$

In Inequalities (19) and (20), $A_d$, $B_d$, $C_d$, $D_d$ are the closed-loop system matrices written by

$$
A_d =\begin{bmatrix}
A & B_d \\
\bar{B}_K C_d & \bar{C}_K
\end{bmatrix}
$$

$$
B_d =\begin{bmatrix}
B_1 \\
\bar{B}_K D_{21}
\end{bmatrix}
$$

$$
C_d =\begin{bmatrix}
C_1 & D_{21} \bar{C}_K
\end{bmatrix}
$$

$$
D_d = D_{11}
$$

In order to convert the inequality in Eq. (20) with $X>0$ into a set of linear matrix inequalities, first, $X$ and $X^{-1}$ are partitioned as
in which the dimensions of $S, N, V, R, M, U$ are all $N \times N$, and $S > 0, R > 0$. For simplicity, the following discussion is confined to the scope that $M$ and $N$ have full column ranks. Using the identity $X^{-1}X = I$ with expression given in Eq. (22) yields the relations $MN^T + RS = I$ and $RN + MV = 0$. Therefore, it can be easily verified that $X$ satisfies

$$X\Pi_1 = \Pi_2$$

with

$$\Pi_1 = \begin{bmatrix} R & I \\ M^T & 0 \end{bmatrix} > 0$$

and

$$\Pi_2 = \begin{bmatrix} I & S \\ 0 & N^T \end{bmatrix} > 0$$

Then, the inequality in Eq. (20) with $X > 0$ is solvable if and only if the following LMI system is feasible; i.e.

$$\begin{bmatrix} R & I \\ I & S \end{bmatrix} > 0$$

and

$$\begin{bmatrix} \Phi_R & \Phi^T_S \\ \Phi & \Phi_S \end{bmatrix} < 0$$

in which

$$\Phi_R = AR + RA^T + B_2^T\hat{C}_K + \hat{C}_K^TB_2^T + \begin{bmatrix} B_1^T \\ C_1R + D_{21}^T\hat{C}_K \\ -D_{11} \end{bmatrix}^{-1} \begin{bmatrix} B_1^T \\ C_1R + D_{21}^T\hat{C}_K \end{bmatrix}$$

and

$$\Phi_S = A^TS + SA + C_2^T\hat{B}_K + B_2^TC_2 + \begin{bmatrix} B_1^T \\ C_1 \end{bmatrix}^{-1} \begin{bmatrix} B_1^T \\ -D_{11} \end{bmatrix}^{-1} \begin{bmatrix} B_1^T \\ C_1 \end{bmatrix}$$

The notation $\cdot^+$ in Inequalities (30) and (31) represents a component equal to the transpose of its symmetric component. Consequently, Inequalities (24), (30), and (31) becomes the set of linear matrix inequalities for the matrices $R, S, \hat{B}_K$, and $\hat{C}_K$.

References


